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Influence of Steady Pitch Rate on 2-D Airfoil Aerodynamic Characteristics at Incidence

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I. Introduction

IN various dynamic problems it is important to know the constituent parts of aerodynamic loads due to pitch rotation. However, the attention given to this question at higher angles of attack with flow separation has been very low. Sometimes the problem of modeling aerodynamic loads due to steady pitch rotation is mixed up with modeling aerodynamic loads due to incidence variation. Some discussions of the question can be found in [1]. These are really two distinct tasks. The first task is a steady one. It is possible to imagine motions with a constant angle of attack and various pitch rates. These are circular orbital motions with various radii. On the other hand a translational motion with zero pitch rate and varying incidence could exist also. In the first case the circulation is constant. In the second case a time-dependent vortex wake exists behind the body. The present work is devoted to some estimations using thin airfoil theory of pitch rate effects on a 2-D airfoil with flow separation at high constant angles of attack.

For low angles of attack the influence of steady pitch rate on airfoil loads was investigated, for example, in [2]. For the case of zero pitch rate the effects of flow separation on aerodynamic loads was investigated using the linearized thin airfoil theory in [3]. This approach was now applied to estimations of pitch rate effects on a 2-D airfoil with flow separation at high constant angles of attack. For comparison, numerical estimations from the well-known XFOIL code [4] were used. The effect of steady pitch rate is simulated using the approximate technique of curved models [5] (pitch rate induced curvature).

II. Thin Airfoil Approximation

One possible approximation for an aerodynamic loads estimation taking account of flow separation is the use of thin airfoil theory with the separated region modeled by a semi-infinite Kirchhoff zone of constant pressure. It has been shown for the static case [3] that if the longitudinal separation point is known (from corresponding boundary layer calculations, for example) then the analytical expressions for normal force and pitching moment coefficients give results very close to numerical calculations. In this Technical Note the same procedure is applied to the case of airfoil with steady pitch rate.

Thus the problem of ideal flow about a 2-D symmetrical thin airfoil is considered. The scheme of the flow is shown in Fig. 1. The airfoil chord is $c = 1$, and in the body fixed coordinate system the flow velocity at infinity is $V_0 = 1$ and is inclined to the longitudinal axis at angle α. The airfoil is rotating with constant pitch rate q about the point x_0. To obtain the nondimensional value of pitch rate q the airfoil semichord and free stream velocity are used. The trajectory of the airfoil in the fixed coordinate system is a part of a circular orbit. On the airfoil upper surface the separation region is defined by the starting point x_s. The boundaries of the separated region are the two free streamlines originating at this point and the trailing edge of the airfoil. The main simplifying assumption is that the separation zone can be also treated as a thin region and the respective boundary conditions can therefore be enforced on the Ox axis. The curvature of the separation region due to airfoil rotation is also neglected.

On the wetted part of the airfoil ($y = +0$, $0 < x < x_s$ and $y = -0$, $0 < x < 1$) the normal velocity is

$$V(x) = -\alpha + 2q(x - x_0)$$

On the separation streamlines $y = +0$, $x_s < x < \infty$ and $y = -0$, $1 < x < \infty$ the disturbance longitudinal velocity is

$$U(x) = 0$$

Using the same technique as in [3] the corresponding boundary-value problem in the complex variable plane $z = x + iy$ can be solved and the pressure distribution along the airfoil obtained in the form of a singular integral. Integration of this pressure distribution results in the expressions for normal force and pitching moment coefficients

$$\begin{aligned} C_N &= \frac{1}{2}\pi\alpha(1+\sqrt{x_s})^2 + \frac{1}{16}\pi q(1+\sqrt{x_s})^2 \\ &\quad \times (15 - 18\sqrt{x_s} + 15x_s - 16x_0) \\ C_m &= -\frac{1}{32}\pi\alpha(1+\sqrt{x_s})^2(5 - 6\sqrt{x_s} + 5x_s - 16x_0) \\ &\quad - \frac{1}{64}\pi q(1+\sqrt{x_s})^2(25 - 80x_0 + 64x_0^2 - 44\sqrt{x_s} \\ &\quad + 96x_0\sqrt{x_s} + 54x_s - 80x_0x_s - 44x_s\sqrt{x_s} + 25x_s^2) \end{aligned} \tag{1}$$

The rotation axis x_0 was used as the reference point during the calculation of the pitching moment coefficient. The above equations give explicit expressions for the dependence of the aerodynamic loads upon the problem parameters α, q, x_s, and x_0.

Using Eq. (1) it is possible to derive expressions for the aerodynamic derivatives C_{N_q} and C_{m_q} which are used for mathematical modeling of steady pitch rotation in various flight dynamics problems if q is not large. Simple analytical calculation gives for the normal force coefficient derivative

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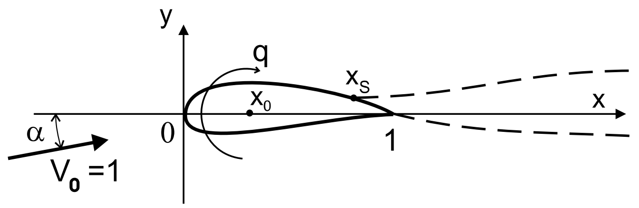

Fig. 1 Schematic for flow about a thin airfoil with upper surface separation with account of steady pitch rate.

$$C_{N_q} = \frac{1}{2}\pi\alpha\frac{(1+\sqrt{x_s})}{\sqrt{x_s}}\frac{dx_s}{dq} + \frac{1}{16}\pi(1+\sqrt{x_s})^2 \times (15 - 18\sqrt{x_s} + 15x_s - 16x_0) \quad (2)$$

and for the pitching moment coefficient derivative

$$C_{m_q} = -\frac{1}{16}\pi\alpha\frac{(1+\sqrt{x_s})}{\sqrt{x_s}}(1 - 4\sqrt{x_s} + 5x_s - 8x_0)\frac{dx_s}{dq} - \frac{\pi}{64}(1+\sqrt{x_s})^2(25 - 80x_0 + 64x_0^2 - 44\sqrt{x_s} + 96x_0\sqrt{x_s} + 54x_s - 80x_0x_s - 44x_s\sqrt{x_s} + 25x_s^2) \quad (3)$$

Equations (2) and (3) show that aerodynamic pitch rotary derivatives depend considerably upon the derivative dx_s/dq, i.e., how the separation starting point is influenced by the steady pitch rotation.

For fully attached flow, with $x_s = 1$, Eq. (1) gives

$$C_N = 2\pi\alpha - 4\pi q\left(x_0 - \tfrac{3}{4}\right)$$
$$C_m = 2\pi\alpha\left(x_0 - \tfrac{1}{4}\right) - 4\pi q\left(x_0 - \tfrac{1}{2}\right)^2 \quad (4)$$

These expressions are well known from classical linear thin airfoil theory [2]. Two consequences of Eq. (4) are that for low angles of attack the normal force coefficient does not depend upon the pitch rate if the airfoil rotates about point $x_0 = 3/4$, and that the pitching moment coefficient does not depend upon the pitch rate if the airfoil rotates about point $x_0 = 1/2$. These facts could be useful to test numerical approaches to the calculation of steady pitch rate effects for 2-D airfoil.

Using Eq. (1) it is possible to obtain an estimation of thin symmetrical airfoil aerodynamic loads for arbitrary values of parameters α, q, and x_0 if the separation starting point coordinate x_s is given. The position of the point x_s where the boundary layer separates depends upon the values of the three above parameters and other factors including Reynolds number, turbulence of incoming flow, etc. In the following section the numerical approach will be used for the estimation of this point position.

III. Numerical Calculations

For numerical estimation of airfoil aerodynamic loads the XFOIL code [4] was used, enabling calculation of the aerodynamic characteristics of airfoils over a wide range of angles of attack, with boundary layer transition and possible flow separation. In this software an ideal fluid panel method is coupled with an integral boundary layer calculation and an envelope e^n transition criteria. XFOIL was developed for the calculation of aerodynamic loads for zero pitch rate, and so to take into account the effect of steady pitch rate it is necessary to reformulate the boundary conditions on the airfoil upper and lower surfaces. In the body fixed frame the normal component of the velocity on the rotating airfoil surface should have (according to [2]) the following form:

$$v_n = U\frac{dy}{ds} - V\frac{dx}{ds} - 2q\left[(x-x_0)\frac{dx}{ds} + y\frac{dy}{ds}\right] \quad (5)$$

where $U = V_0\cos\alpha$ and $V = V_0\sin\alpha$ are the freestream velocity components, q is the steady pitch rate about point x_0, and $y(s)$ and $x(s)$ are the parametrically defined airfoil shape (s is the arc length of the airfoil surface).

The motion of the airfoil with small q will be considered, so that the pitch rate effects can be simulated by a small distortion of the original airfoil shape. The problem is stated as follows: what shape should the airfoil have, which in rectilinear motion will have the boundary conditions equivalent to the boundary conditions of the original airfoil in motion with steady pitch rotation? Let the shape of the distorted airfoil be $y' = y(s) + \delta(s)$ and $x' = x(s)$. Because this airfoil is moving rectilinearly the boundary condition for its normal velocity is

$$v_n = U\frac{dy'}{ds} - V\frac{dx'}{ds} = U\left(\frac{dy}{ds} + \frac{d\delta}{ds}\right) - V\frac{dx}{ds} \quad (6)$$

The normal velocities on both airfoils should be equal. Then from Eqs. (5) and (6) with the approximate assumption $U \approx V_0 = 1$ it follows that in nondimensional form

$$\frac{d\delta}{ds} = -2q\left[(x-x_0)\frac{dx}{ds} + y\frac{dy}{ds}\right] \quad (7)$$

It is evident that the surface distortion function should have the following form:

$$\delta = -q[(x-x_0)^2 + y^2] \quad (8)$$

Using this distortion function the surface of the original airfoil can be changed to simulate the pitch rate effects for given q and x_0. Figure 2 shows the shapes of airfoils obtained from a NACA 0012 original for rotation centers $x_0 = 0.25$ (upper picture) and $x_0 = 0.75$ (lower picture), and for pitch rates $q = 0$ (original shape), $q = \pm 0.025$ and $q = \pm 0.05$.

Thus using the curved airfoils it is possible to estimate steady pitch rate effects by ordinary XFOIL calculations. Of course this technique is an approximate one. For example, the effects of rotation were not taken into account in boundary layer calculations on the airfoil, or on the viscous wake. The present work could be a starting point for future more accurate and more complex calculations of rotary effects.

IV. Calculation Results

Using XFOIL the aerodynamics loads for all airfoils shown in Fig. 2 were calculated for a wide range of angles of attack. The calculations were executed for Mach number $M = 0.15$ and Reynolds number $Re = 1 \times 10^6$. For turbulent transition calculation the recommended value $N_{crit} = 9$ was used.

Figure 3 shows the results obtained for a NACA 0012 airfoil rotated about the point $x_0 = 0.25$. The XFOIL data are shown by various markers for various pitch rates. In the upper picture the

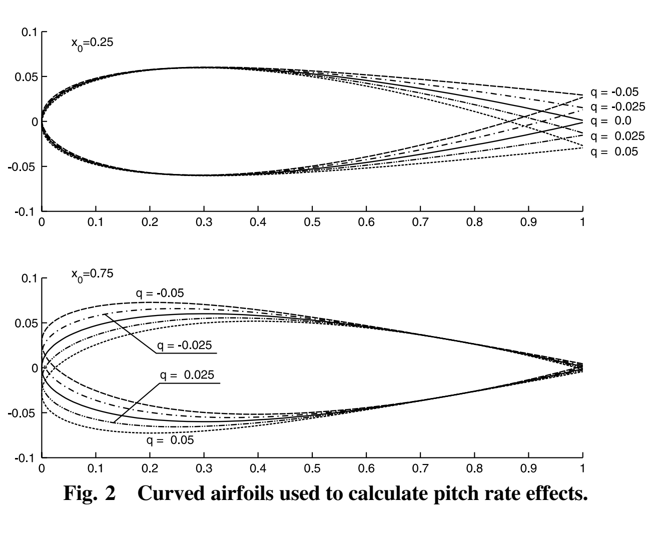

Fig. 2 Curved airfoils used to calculate pitch rate effects.

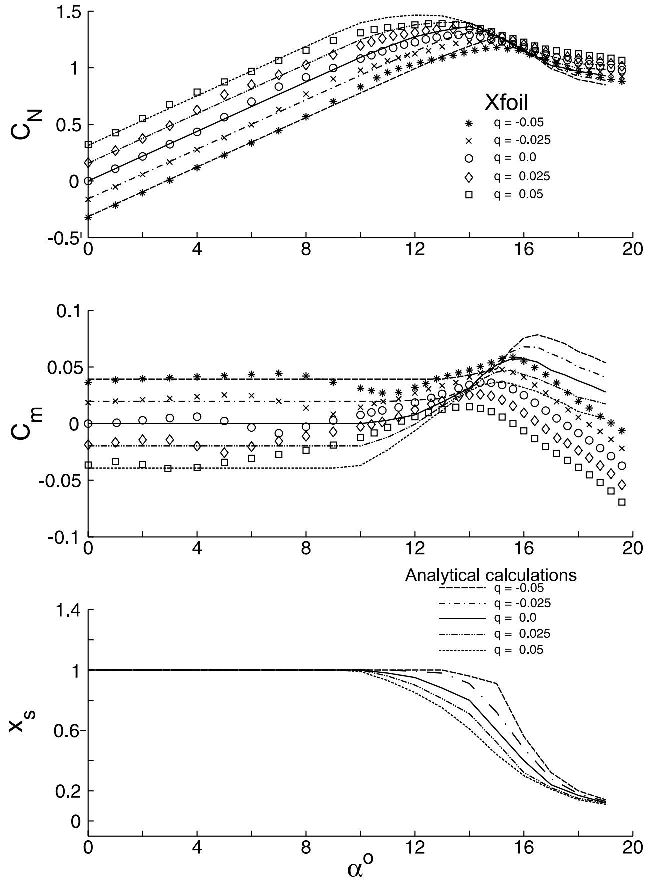

Fig. 3 Influence of steady pitch rate q about $x_0 = 0.25$ on aerodynamic characteristics NACA 0012 airfoil.

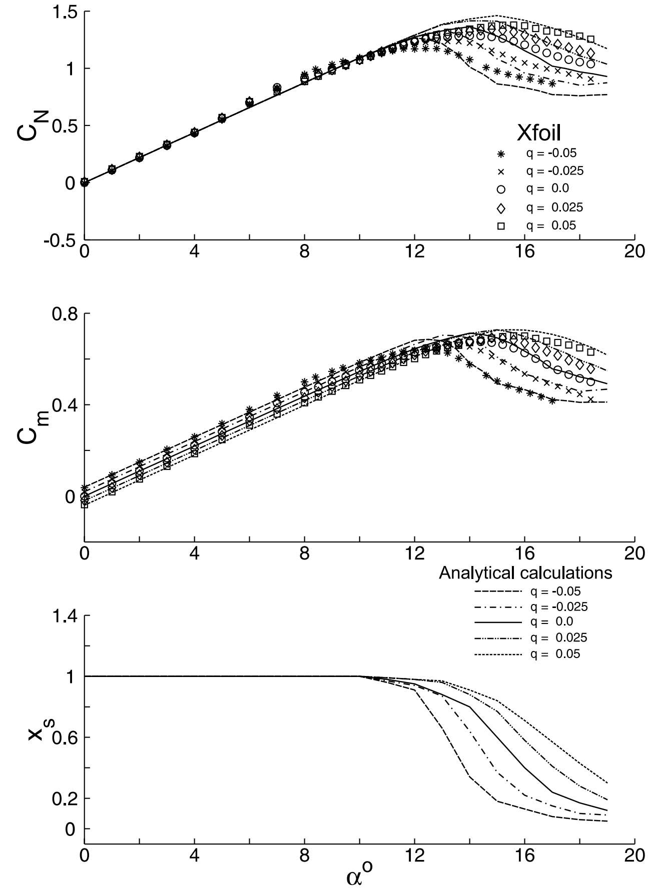

Fig. 4 Influence of steady pitch rate q about $x_0 = 0.75$ on aerodynamic characteristics NACA 0012 airfoil.

results for the normal force coefficient are shown. In the middle part the pitching moment results are presented. In the lower part the corresponding separation point dependencies $x_s(\alpha)$ for various values of q are demonstrated. These dependencies of the separation point location upon angle of attack were estimated by finding points where the skin-friction coefficient C_f vanishes according to XFOIL's integral boundary layer calculations. It can be seen that for this center of rotation positive (nose up) pitch rate results in earlier separation development, whereas negative (nose down) pitch rate results in the separation developing at higher angles of attack in comparison with the airfoil in rectilinear motion. Using the interpolated dependencies $x_s(\alpha)$ the aerodynamic characteristics $C_N(\alpha)$ and $C_m(\alpha)$ could be obtained with the use of the analytical expressions from Eq. (1). The corresponding results are shown as lines of various types in Fig. 3. It is seen that for low angles of attack there is excellent agreement between the XFOIL calculated data and the analytical results. At high angles of attack the agreement is not so good but the qualitative similarity is evident.

In Fig. 4 similar results for the NACA 0012 airfoil rotated about the point $x_0 = 0.75$ are presented. It can be seen that for this center of rotation positive pitch rate results in delayed flow separation (that is, at higher angles of attack) and negative steady pitch rotation results in flow separation at lower α, so that the inverse relationship is obtained in comparison with the forward center of rotation $x_0 = 0.25$. It should be noted that for this center of rotation for low angles of attack the normal force coefficient does not depend upon the pitch rotation value q in accordance with Eq. (4). For this center of rotation there is also a rather good agreement between calculated and analytical results in the whole range of investigated angles of attack.

Using the above results it is interesting to obtain an estimation of the rotary derivatives C_{N_q} and C_{m_q} which are usually used for mathematical modeling of pitch rate effects in flight dynamics. Utilization of the $x_s(\alpha)$ dependencies for various pitch rates q enables an estimation of the derivative $\mathrm{d}x_s/\mathrm{d}q$, as shown in Fig. 5 for

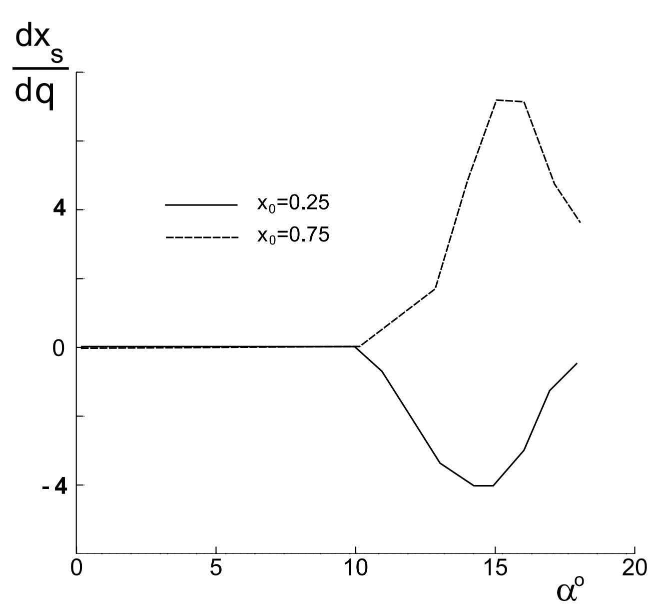

Fig. 5 Estimation of the derivative $\mathrm{d}x_s/\mathrm{d}q$ using XFOIL numerical results.

various centers of airfoil rotation. Using the obtained dependency $\mathrm{d}x_s/\mathrm{d}q(\alpha)$ the derivatives are given by Eqs. (2) and (3). Direct numerical estimations of the derivatives C_{N_q} and C_{m_q} were also made using the XFOIL results $C_N(\alpha)$ and $C_m(\alpha)$ calculated for $q = \pm 0.025$. The data obtained for C_{N_q} and C_{m_q} using both approaches (XFOIL and analytical) are presented in Fig. 6. It can be seen that flow separation at high angles of attack results in a

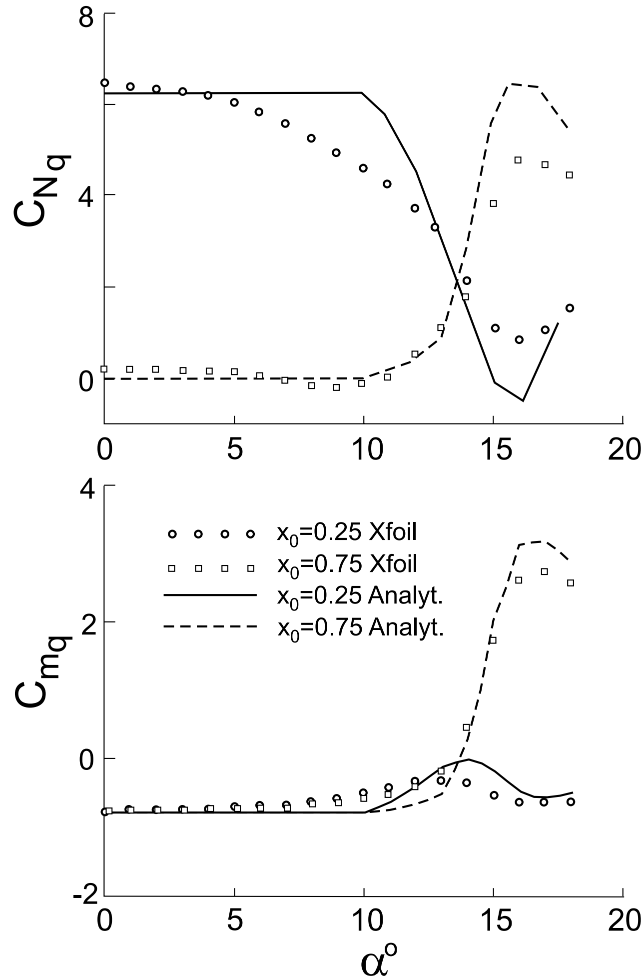

Fig. 6 XFOIL numerical results and analytical estimations for aerodynamic derivatives C_{N_q} and C_{m_q}.

considerable nonlinear variation of the airfoil rotary derivatives C_{N_q} and C_{m_q}. The regions of positive damping ($C_{m_q} > 0$) could be attributed to this effect. It can also be noted that the analytical and numerical calculated results are in satisfactory agreement for both derivatives and various centers of rotation.

V. Conclusions

An investigation of steady pitch rate effects on nonlinear dependencies of 2-D airfoil aerodynamic loads at high angles of attack was undertaken. The linearized thin airfoil theory results with account of flow separation using Kirchhoff's zone of constant pressure were compared with numerical results from XFOIL. For numerical calculations the approximate concept of pitch induced curved airfoil shapes was used. It was found that the steady pitch rate effects have considerable influence on the development of the airfoil upper surface flow separation. It is essential to take into account not only the pitch rate value but the position of the rotation center also. The analytical and numerical estimations of pitch rotary derivatives C_{N_q} and C_{m_q} were obtained. The flow separation effects result in considerable nonlinear variations of these derivatives at high angles of attack.

The results presented here are for a steady-state pitching motion, and so do not take into account any time-history effects. However, by establishing a static relationship between pitch rate and separation point position, this analysis provides the necessary basis for modeling the effects of the pure pitch component on dynamic stall of oscillating airfoils using (for example) the state-space methodology described in [6].

Acknowledgment

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References

[1] Greenwell, D. I., "A Review of Unsteady Aerodynamic Modelling for Flight Dynamics of Manoeuvrable Aircrfat," AIAA Paper 2004-5276, 16–19 Aug. 2004.

[2] Sedov, L. I., *Two-Dimensional Problems in Hydrodynamics and Aerodynamics*, Wiley, New York, 1965, Chap. 2.

[3] Khrabrov, A., and Ol, M., "Effects of Flow Separation on Aerodynamic Loads in Linearized Thin Airfoil Theory," *Journal of Aircraft*, Vol. 41, No. 4, 2004, pp. 944–948.

[4] Drela, M., "XFOIL: An Analysis and Design System for Low Reynolds Number Airfoils," *Conference on Low Reynolds Number Airfoil Aerodynamics*, Univ. of Notre Dame Press, South Bend, IN, June 1989.

[5] Gurjenko, G. A., "Twisted Model Technique and its Application to Rotary Motion of Aeronautical Vehicles," *TsAGI Transactions*, No. 182, 1934 (in Russian).

[6] Goman, M. G., and Khrabrov, A. N., "State-Space Representation of Aerodynamic Chracteristics of an Aircraft at High Angles of Attack," *Journal of Aircraft*, Vol. 31, No. 4, 1994, pp. 1109–1115.